Predictions from Lattice QCD

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USQCD All Hands' Meeting

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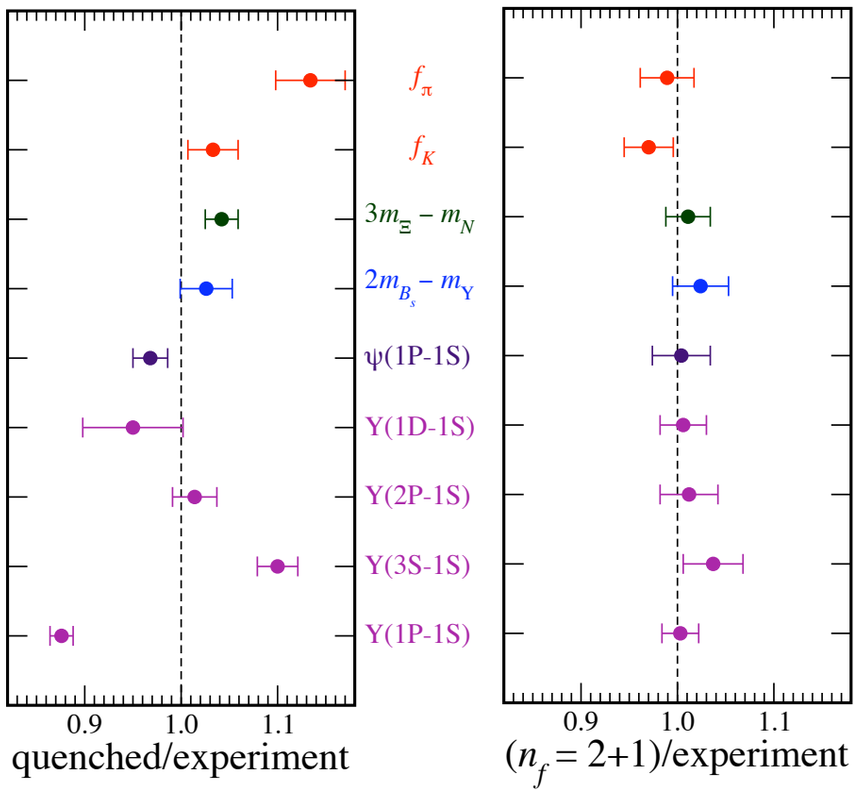
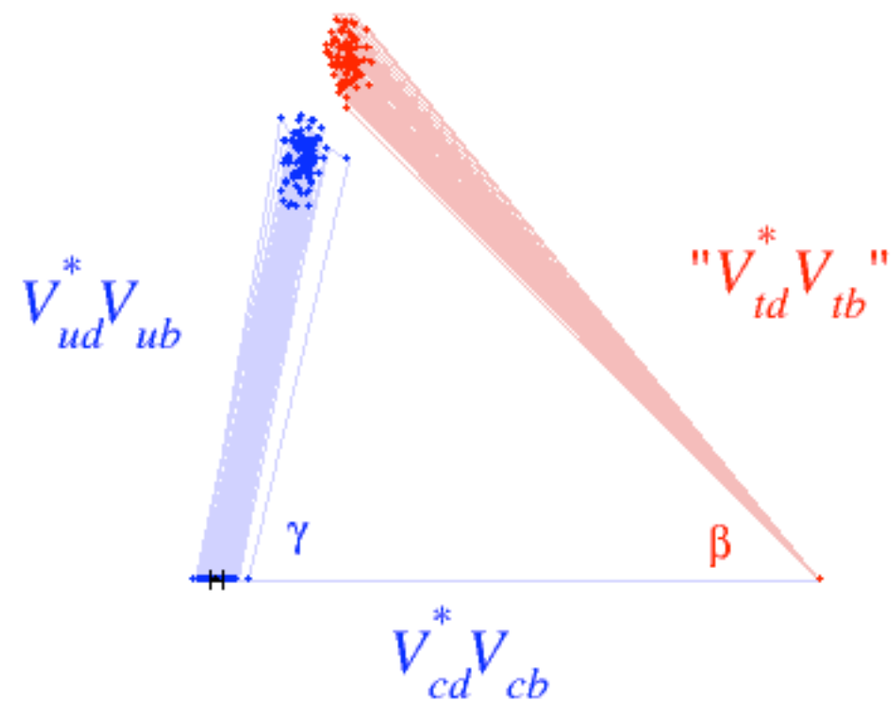
Outline

- Semileptonic *D* decays [hep-ph/0408306 → PRL]
  - Prototype for semileptonic *B* decay & $|V_{ub}|$
- Leptonic *D* Decay [hep-lat/0410030 ← Lat '04]
  - Prototype for B-$\overline{B}$ mixing & $|V_{td}|$
- Mass of the B_c Meson [hep-lat/0411027 → PRL]
  - Test of heavy-quark discretization effects

Preliminaries

Context



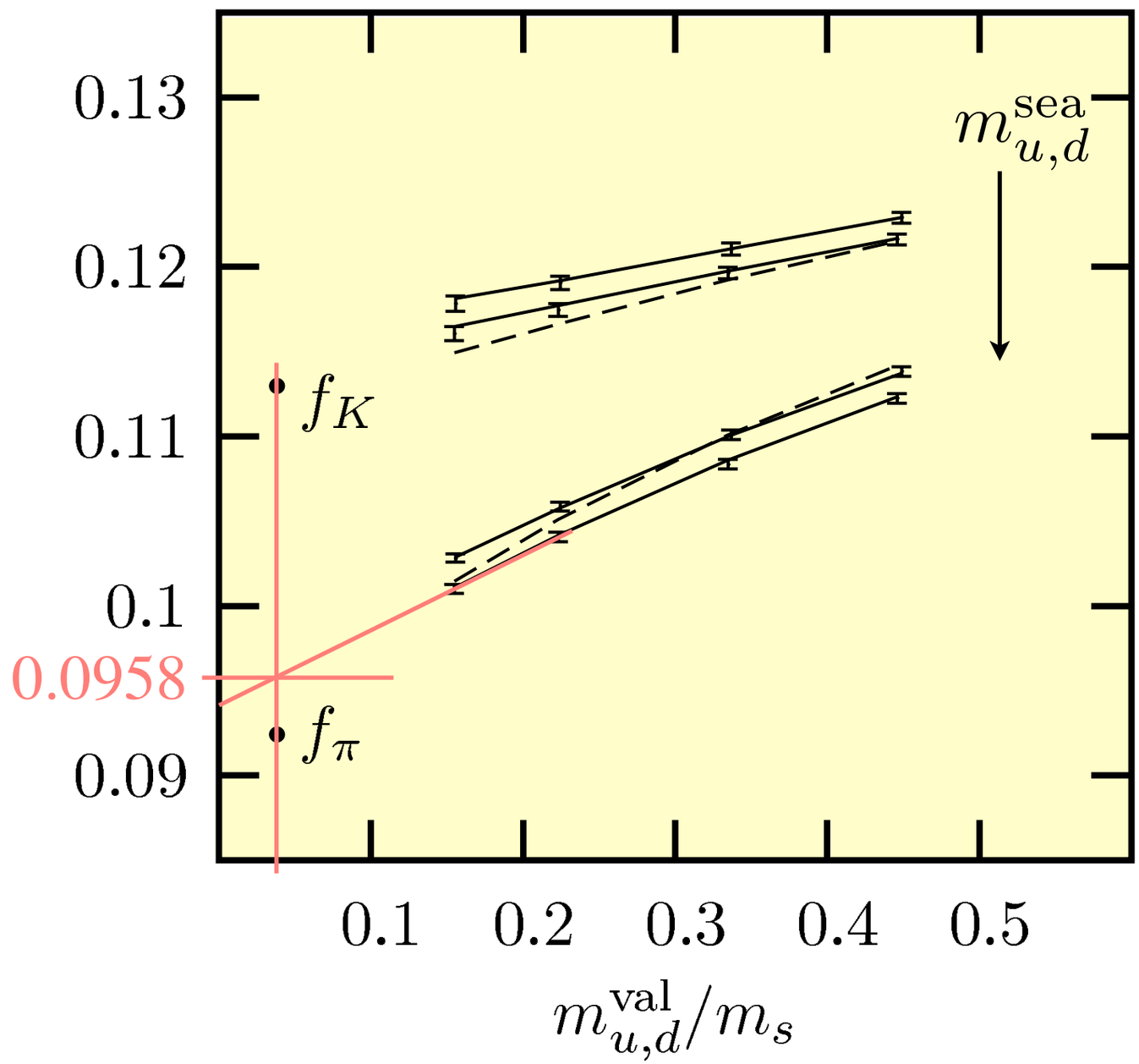


- Recent progress looks dramatic, and the goals are bold.
- This talk: the first steps.

Unquenched Calculations

- Results assume (and suggest!?)
  - $[\det_4 M]^{1/4} \doteq \det_1(\not{D} + m)$
  - staggered (partially quenched) chiral PT
  - effective field theories for heavy quarks
- Predictive tests of these assumptions valuable.
- Even without "issues," outsiders like tests of inscrutable black boxes.

Chiral Extrapolation



- Dots at 0.04 are PDG.
- Error bars are lattice QCD.
- Linear extrap (by eye).
- Gasser-Leutwyler χlog gets closer (solid).
- Sharpe-Shoresh χlog even closer (dashed).
- Aubin-Bernard χlog has better CL.

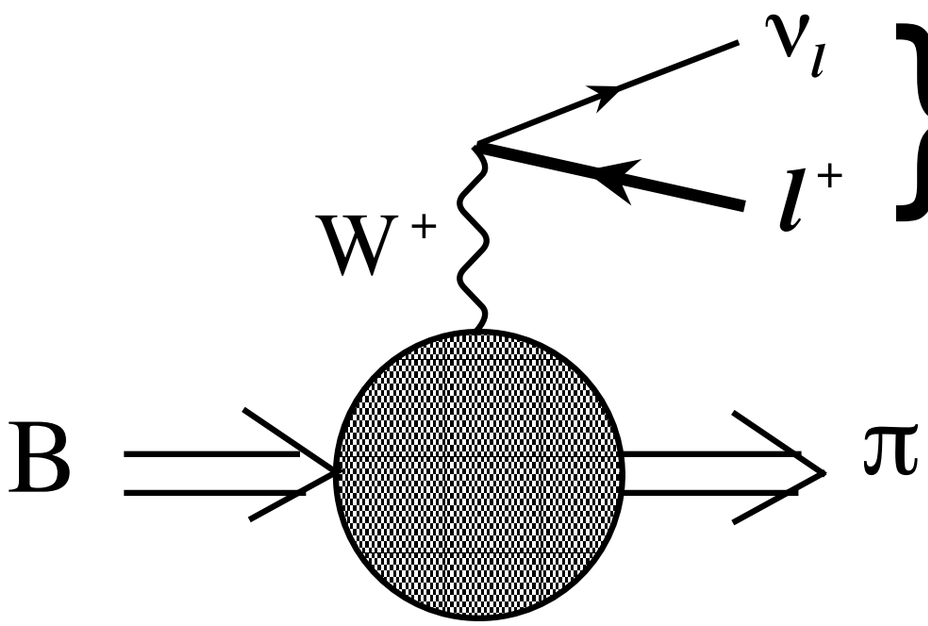
Tests

calculation	*light sea*	χ*PT*	*heavy*
semileptonic	★★	★	★
leptonic	★★	★★★	★
B_c mass	★★	★	★★★

Semileptonic *D* Decay

$$f_+^{D\to\pi}(q^2)\ \&\ f_+^{D\to K}(q^2)$$

Semileptonic Decay



$$q^2 = m_B^2 + m_\pi^2 - 2m_B E$$

$$\frac{d\Gamma}{dE} = \frac{G_\mu^2 m_B}{12\pi^3}|V_{ub}|^2 p^3 |f_+(E)|^2$$

$$\langle \pi(p_\pi)|\mathcal{V}^\mu|B(p_B)\rangle = f_+(E)\left[p_B + p_\pi - \frac{m_B^2 - m_\pi^2}{q^2}q\right]^\mu + f_0(E)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

Pology

- For $E < 0$, there are poles and cuts, and so on, from real states in $B\pi$ ($D\pi$) scattering.
  - vector mesons for f_+ at $E \sim -\Lambda^2/m_Q, -\Lambda$ ...
  - scalar mesons for f_0 at $E \sim -\Lambda$ ...
- Their effects spill into physical region $E > 0$.
- For f_+, the B^* (D^*) is nearby.

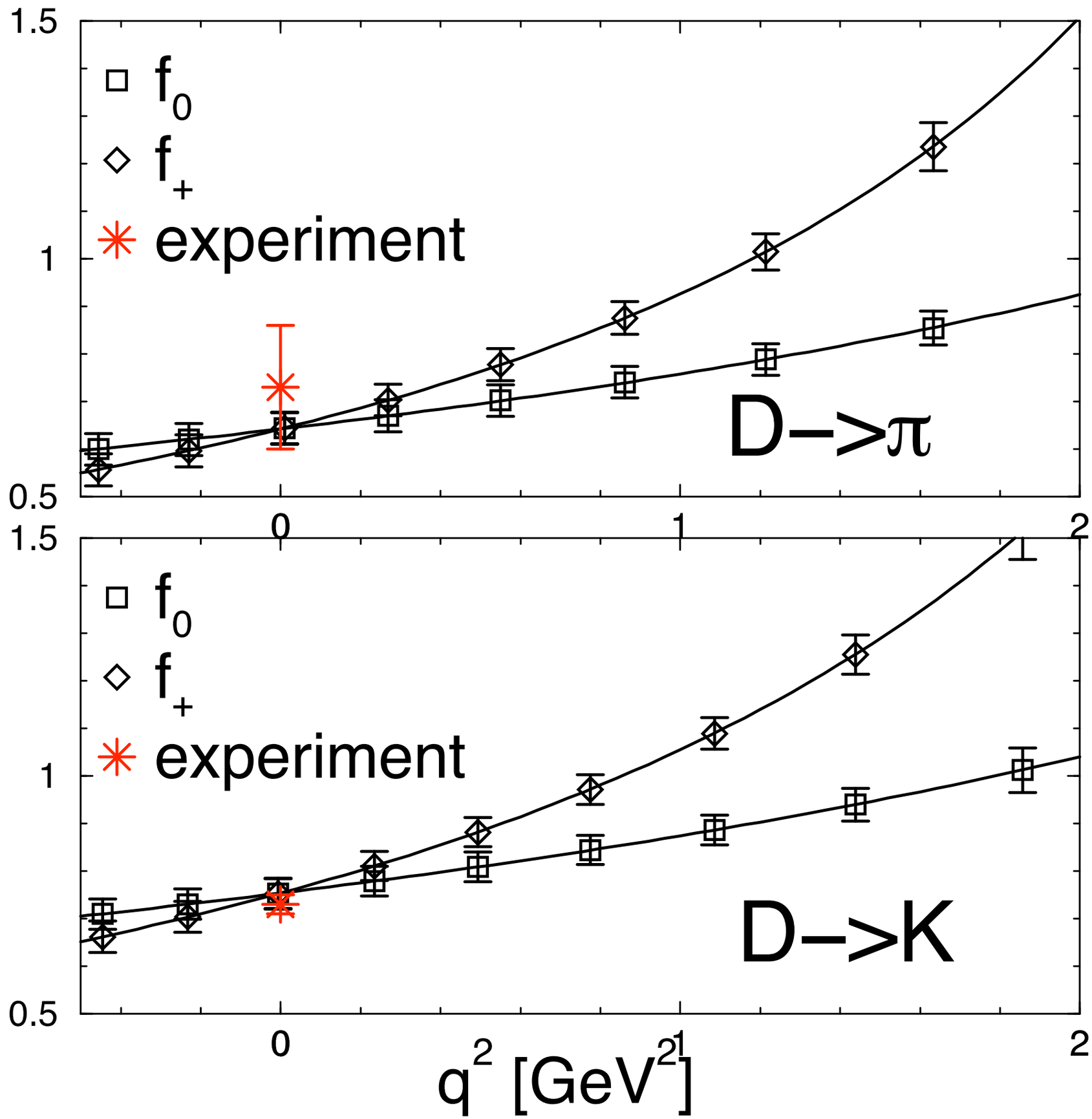
BK *Ansatz*

- With this in mind Becirevic and Kaidalov proposed the parametrization

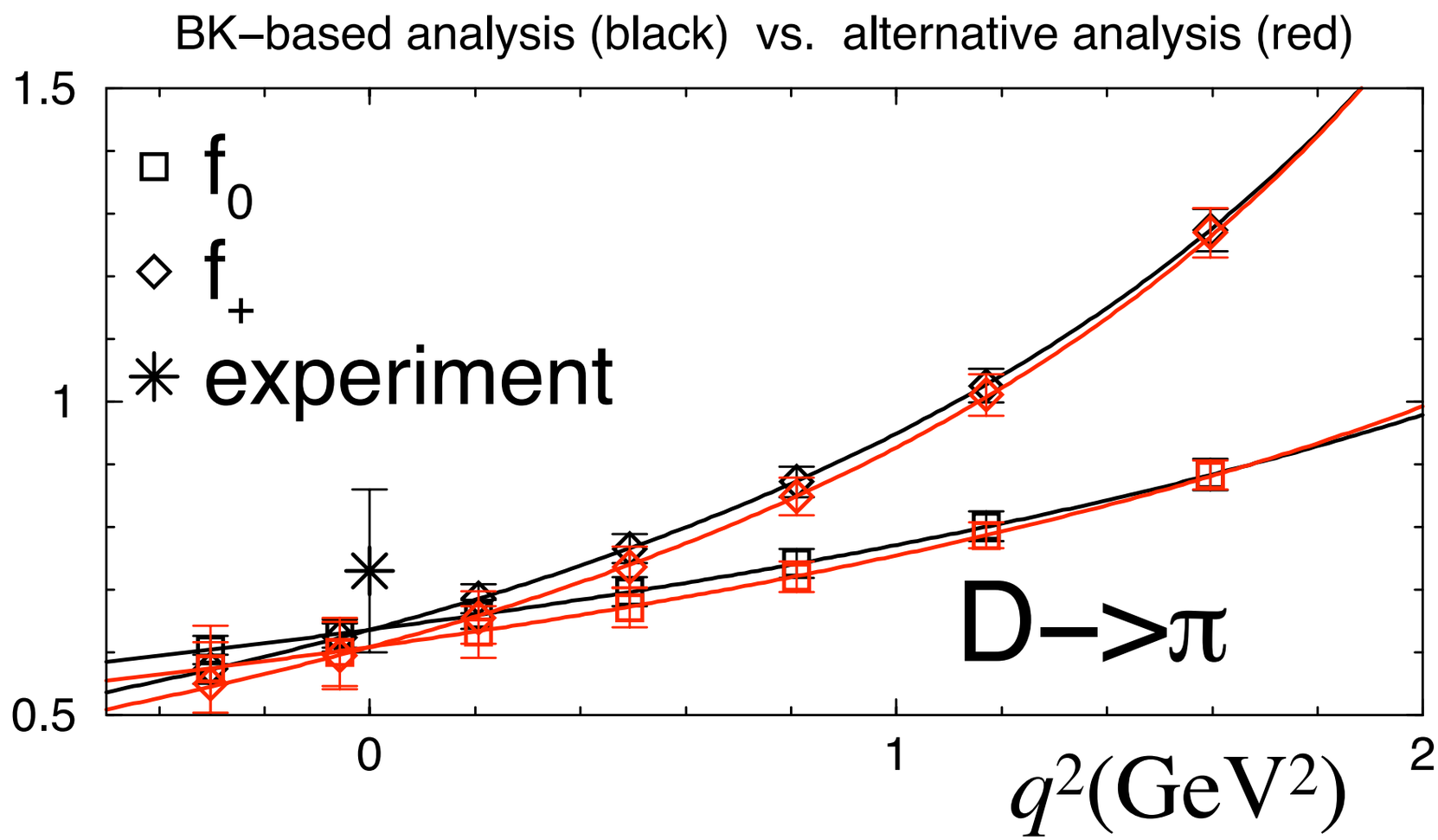
$$f_+(q^2) = \frac{F}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)},$$
$$f_0(q^2) = \frac{F\beta}{\beta-\tilde{q}^2}, \qquad \tilde{q}^2 := q^2/m_{D^*}^2$$

- Builds in kinematic constraint, the closest pole in f_+, and has parameters for the slop.

- Advantages
  - builds in pole, & also heavy-quark scaling laws
  - fit to BK is most sensitive to low energy, yet f_0 influences f_+ through $F = f_0(0) = f_+(0)$.
- Disadvantages
  - parametrization deteriorates with E
  - fit to BK is sensitive mostly to low energy, and f_0 determines $F = f_0(0)$.


1.5
1
0.5
0
1
2
f0
f+
experiment
D->π
1.5
1
0.5
0
1
2
f0
f+
experiment
D->K
q2 [GeV2]

- An alternative is to avoid BK altogether, and use χPT to extrapolate jointly in (m_q, E):



- Consistent, but no-BK has larger error in low q^2 (high E) region.

hep-ph/0408306

dominant error:
heavy quark
discretization

- $D \to Kl\nu$:

$$f_+^{D\to K}(0) = 0.73(3)(7)$$

$$f_+^{D\to K}(0) = 0.78(5)$$ [BES, hep-ex/0406028]

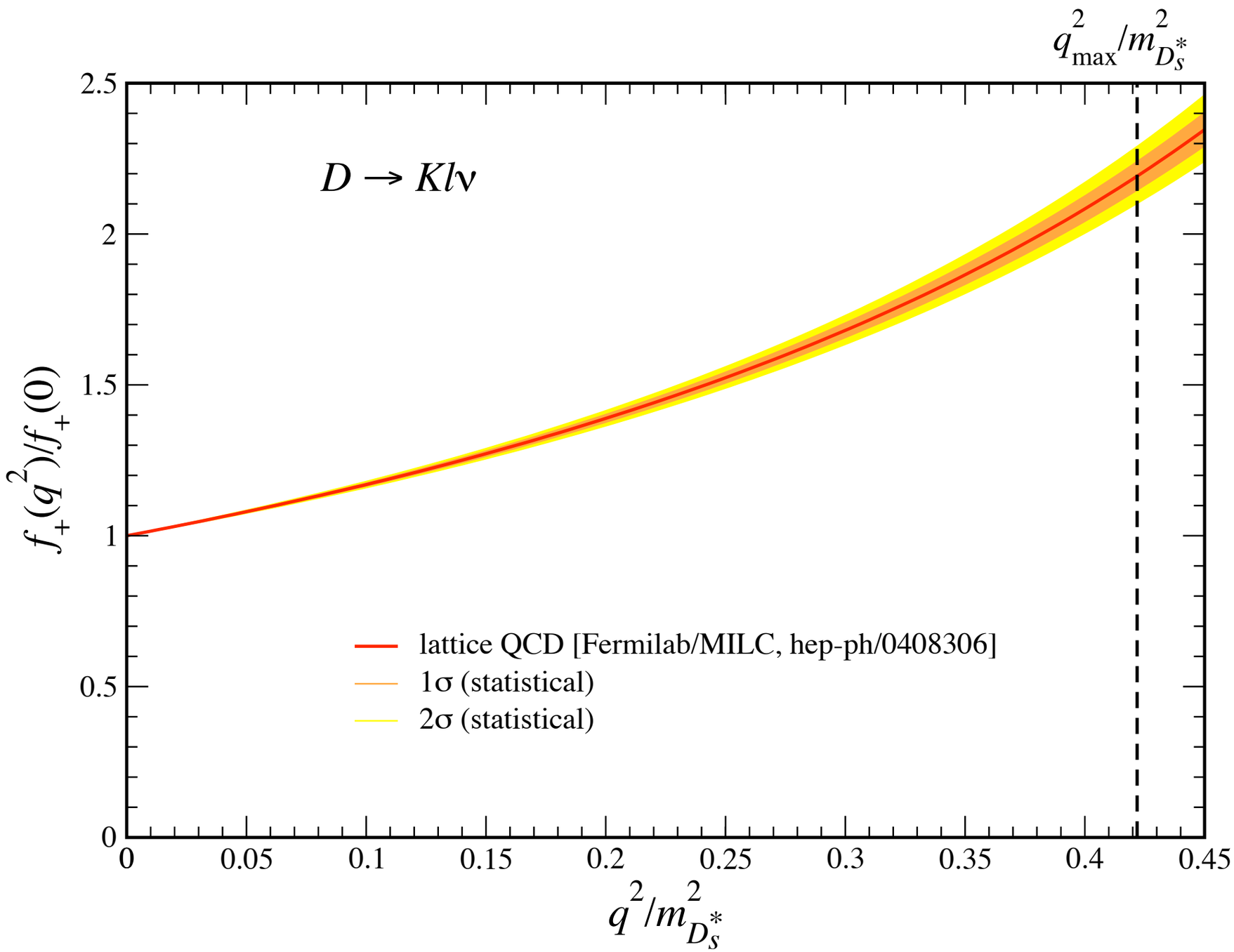
- $D \to \pi l\nu$:

$$f_+^{D\to \pi}(0) = 0.64(3)(6)$$

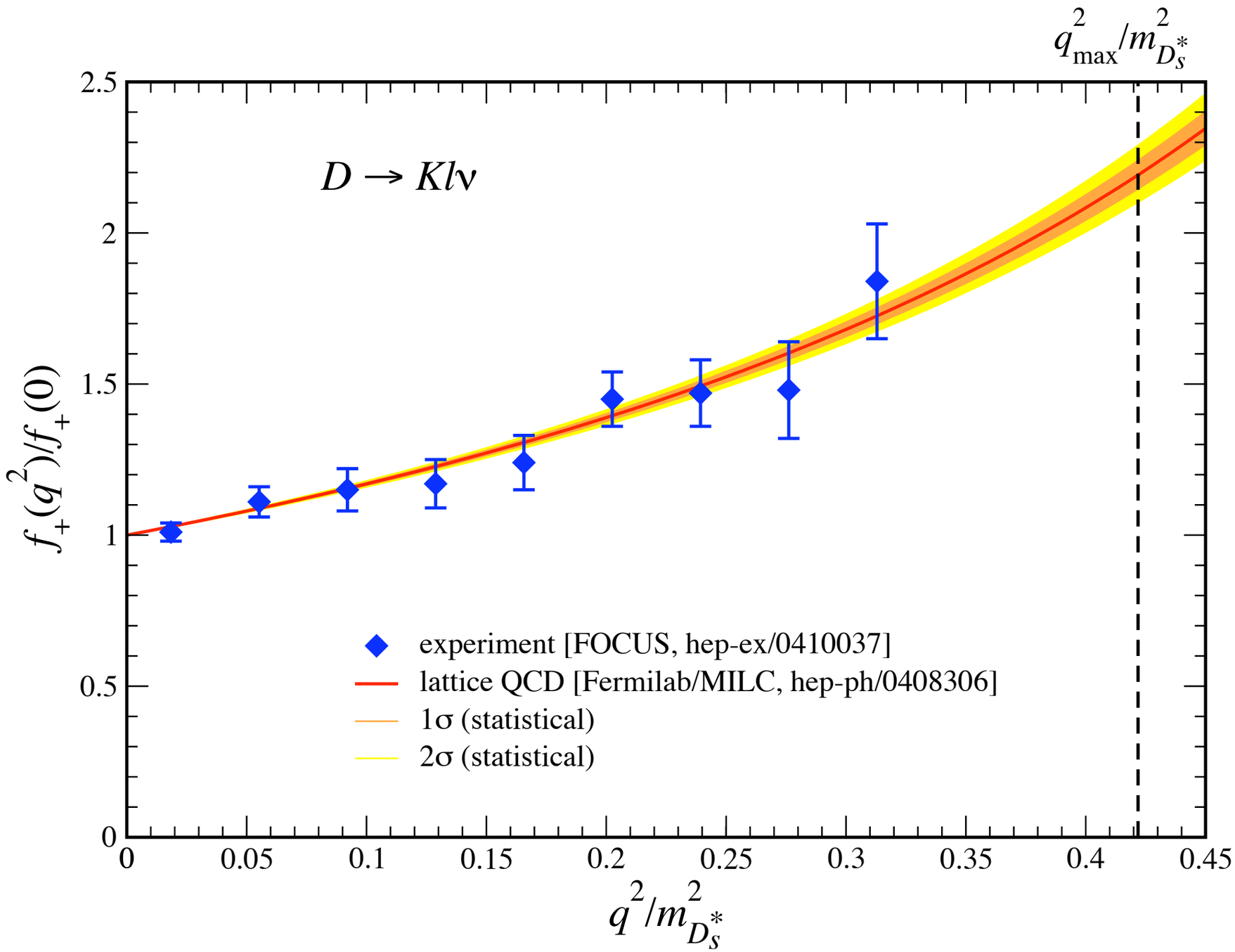
$$f_+^{D\to \pi}(0) = 0.87(3)(9) f_+^{D\to K}$$

$$f_+^{D\to \pi}(0) = 0.86(9) f_+^{D\to K}$$ [CLEO, hep-ex/0407035]

$D \to Kl\nu$ vs. q^2



$D \to Kl\nu$ vs. q^2



All of CKM

- Okamoto has combined our (*i.e.*, his) calculations of $D \to \pi$ and $D \to K$ with preliminary calculations of $B \to D$ to obtain the middle row of the CKM matrix.
  - add $B \to \pi$, $K \to \pi$ to get 5/9, and unitarity to fill out the top row, the right column, and $|V_{ts}|$.
  - add $\sin(2\beta)$ to get the last element, $|V_{td}|$.

hep-lat/0412044

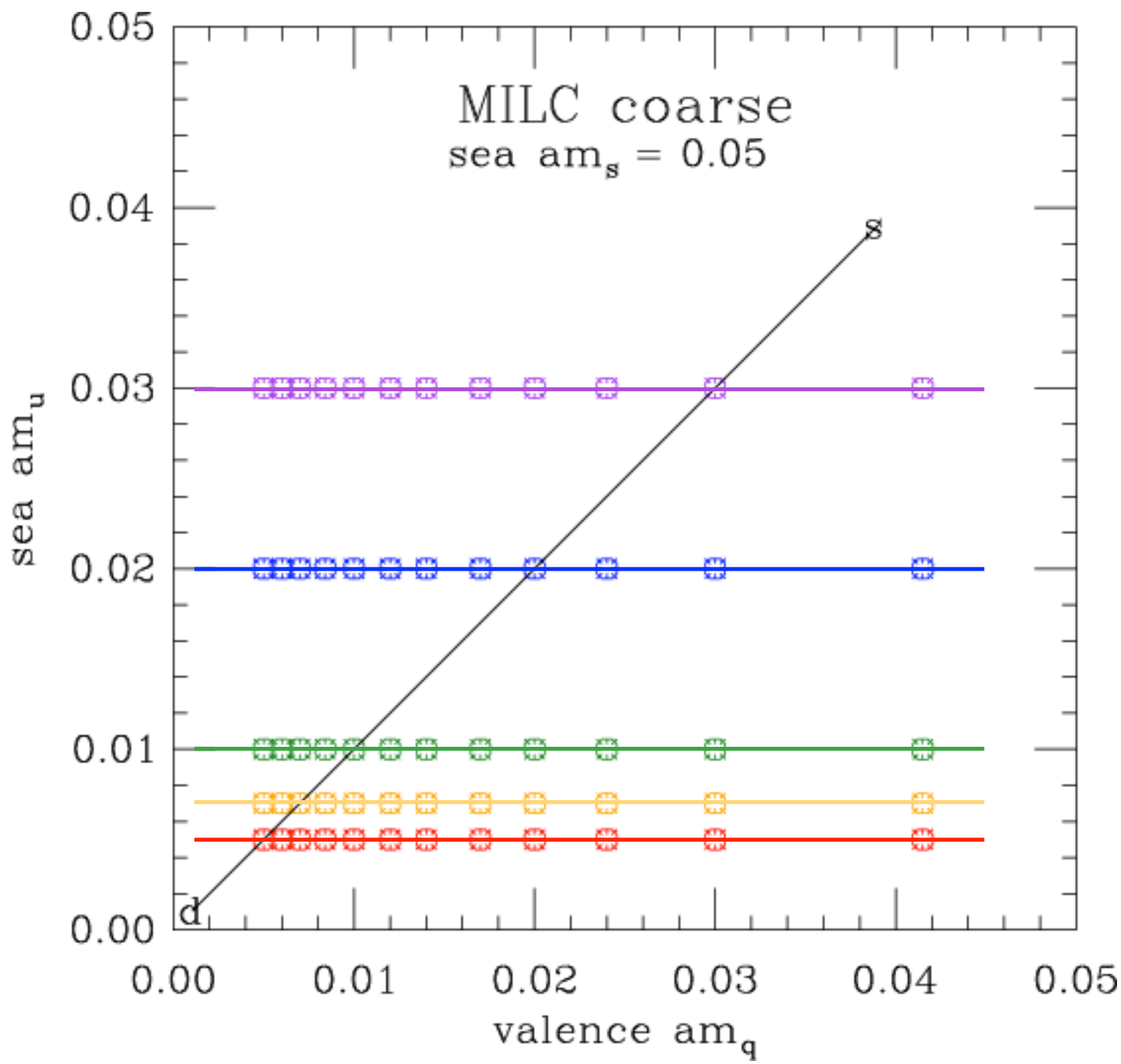
Leptonic *D* Decay

f_{D_s} & f_D

f_{D_s} & f_D

- *D* meson decay constants either
  - determine $|V_{cs}|$ and $|V_{cd}|$
  - check QCD (with $|V_{cs}|$ and $|V_{cd}|$ from CKM unitarity).
- CLEO-*c* is measuring them.
- A test of light quarks and (staggered) PQχPT.

Chiral Extrapolation f_D



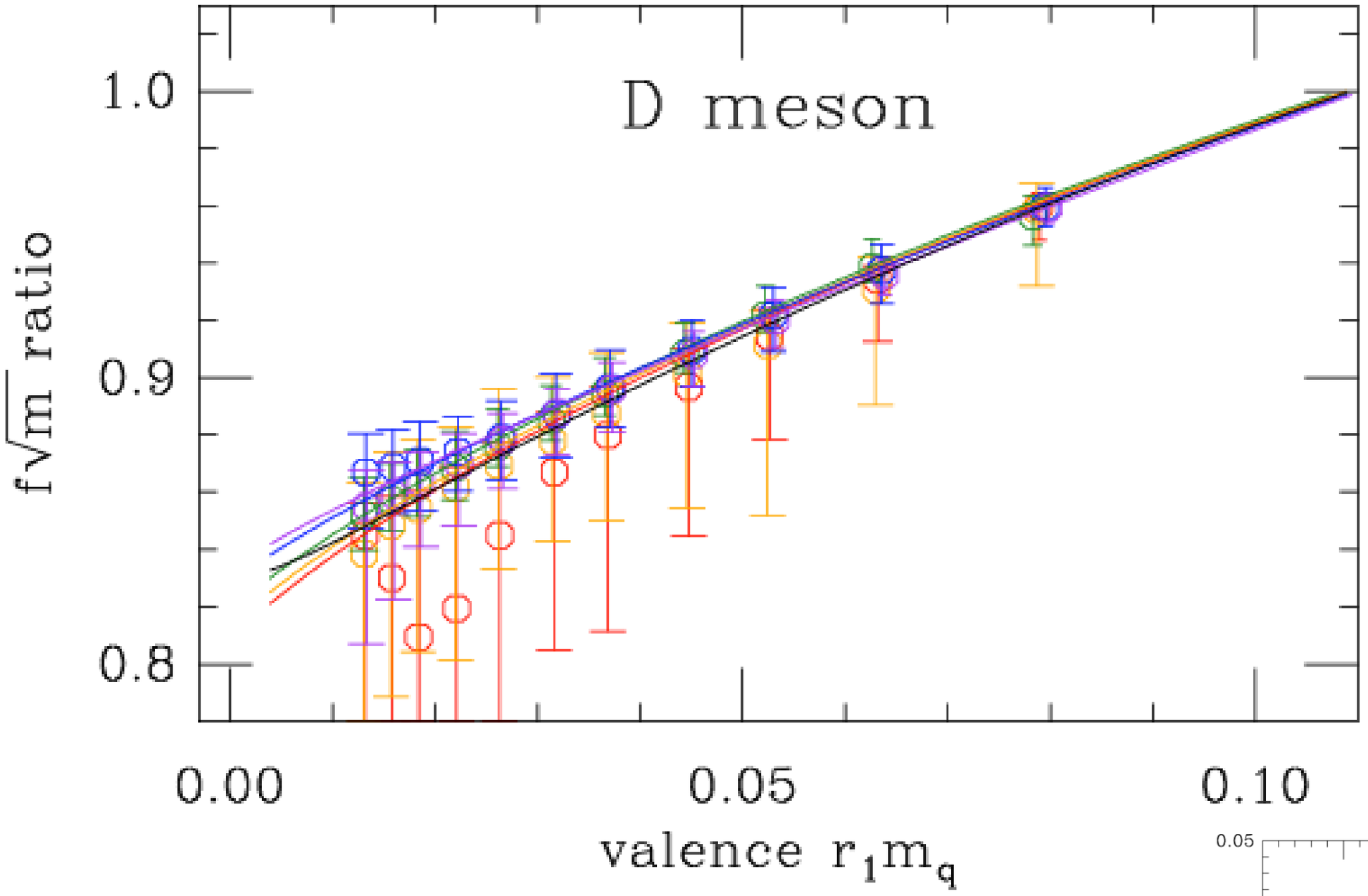
- Extrapolate in sea m_u and valence m_q to get down to real m_l.
- Single fit to all data constrains χPT better.

Staggered PQχPT

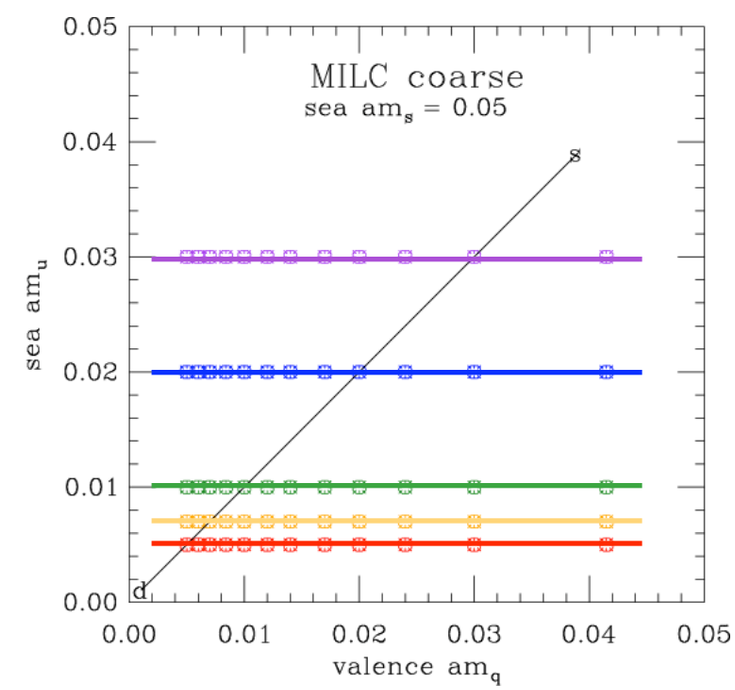
- In the case of decay constants, chiral logs are important.
- In staggered PQχPT, Aubin & Bernard find

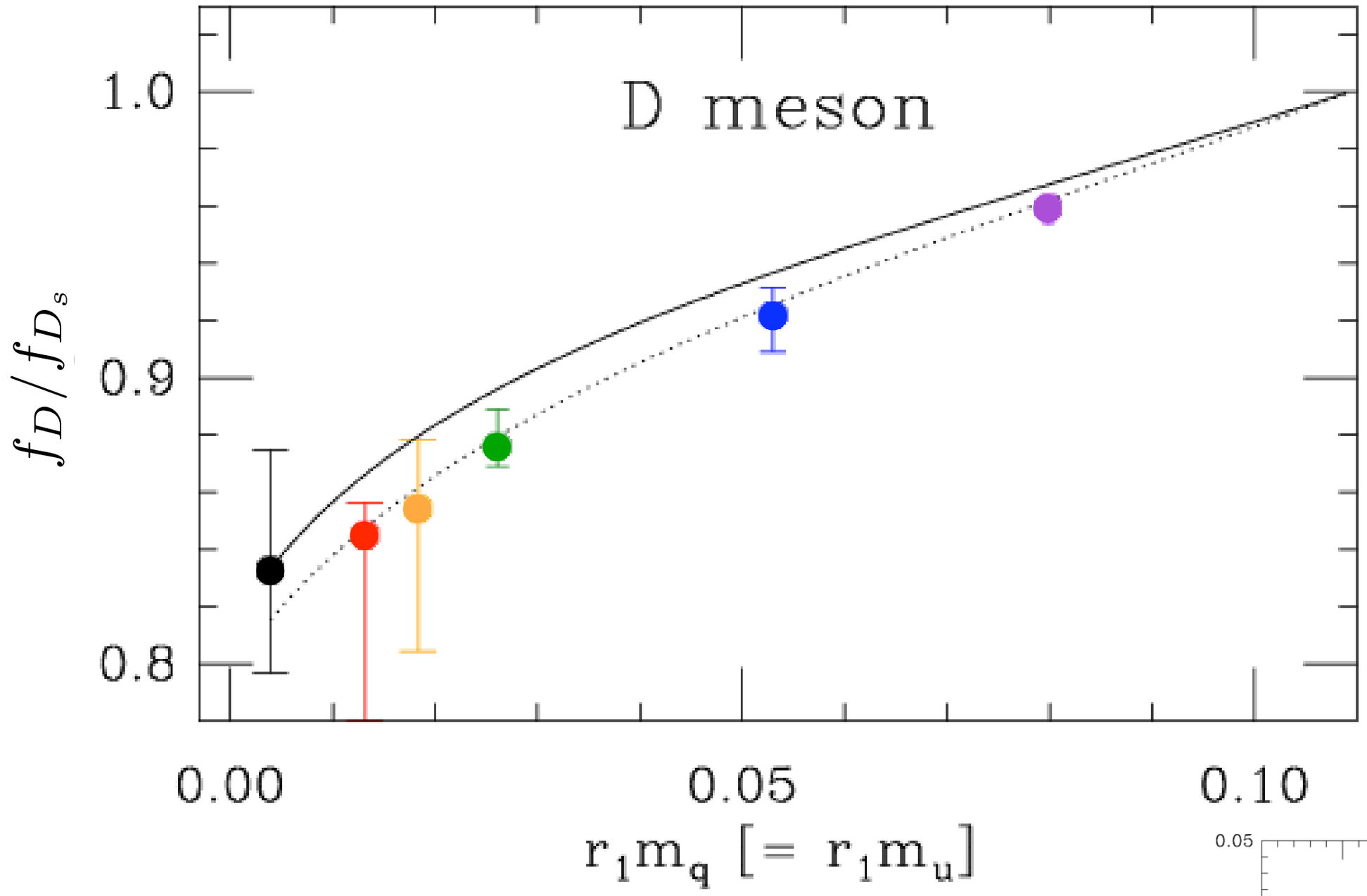
$$m_{uu}^2 \ln m_{qq}^2 \rightarrow \begin{cases} m_{uu}^2 \ln m_{\text{average}}^2 \\ m_{uu}^2 \ln m_{\text{taste singlet}}^2 \end{cases}$$

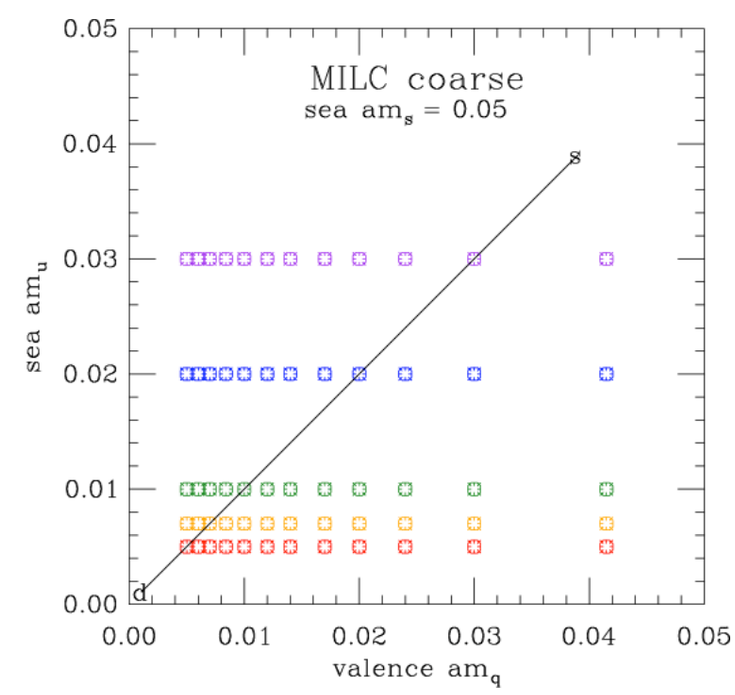
so singularity of PQχPT softened.



Fit all 60 combinations of (am_u, am_q).

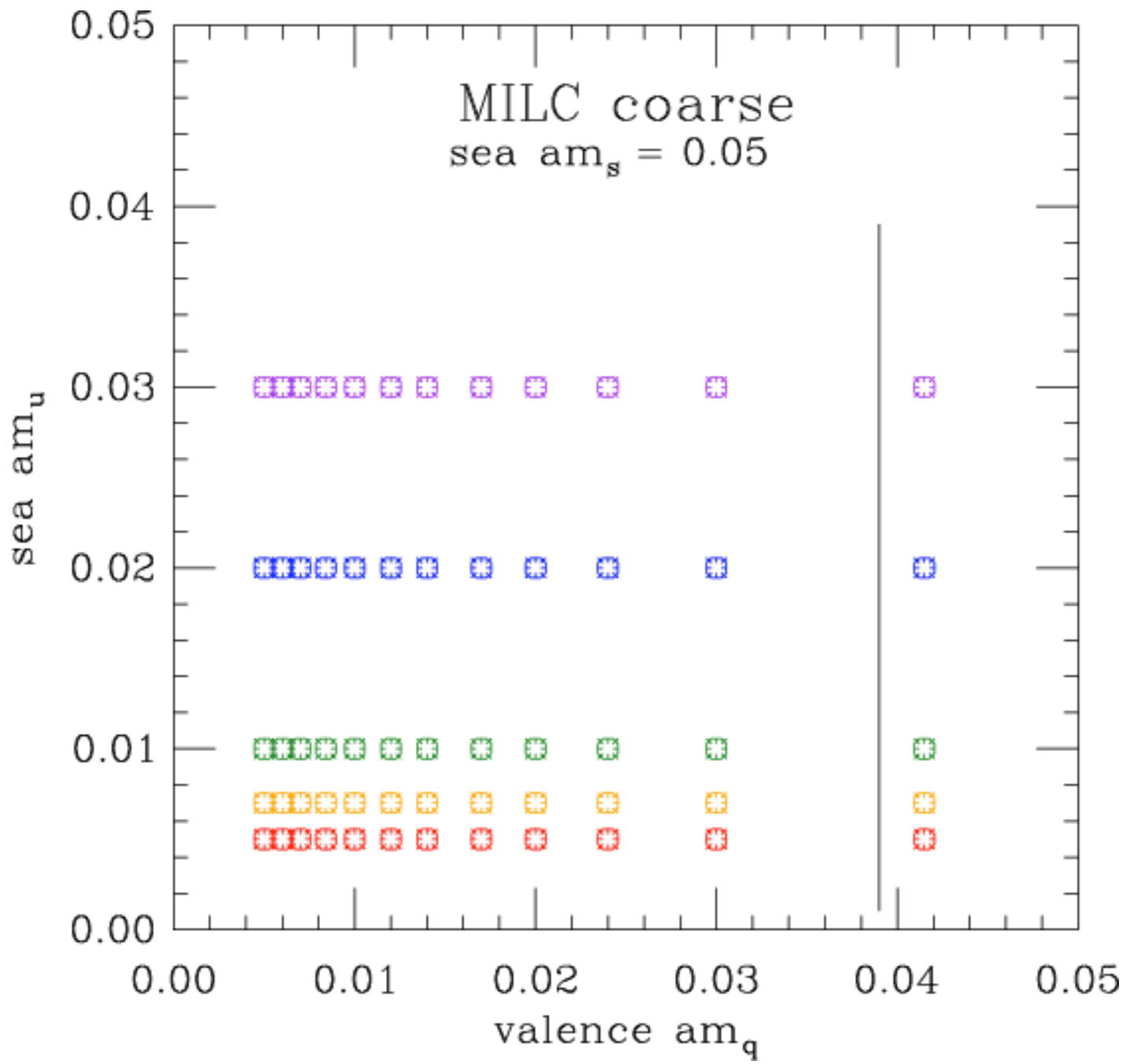




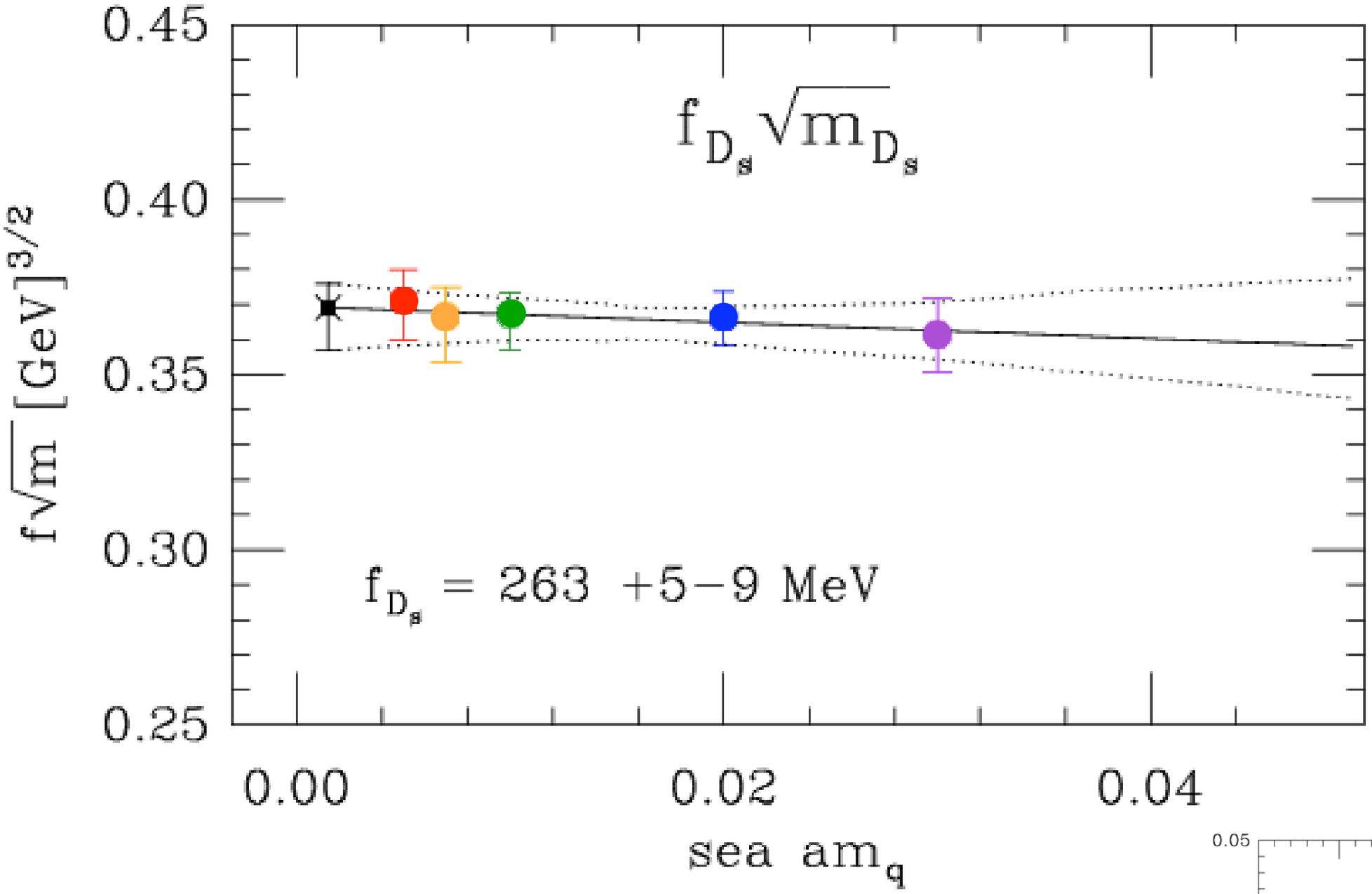


$am_u = am_q$ part
of stagPQχPT fit (dotted)
with $O(a^2)$ bits turned off (solid)

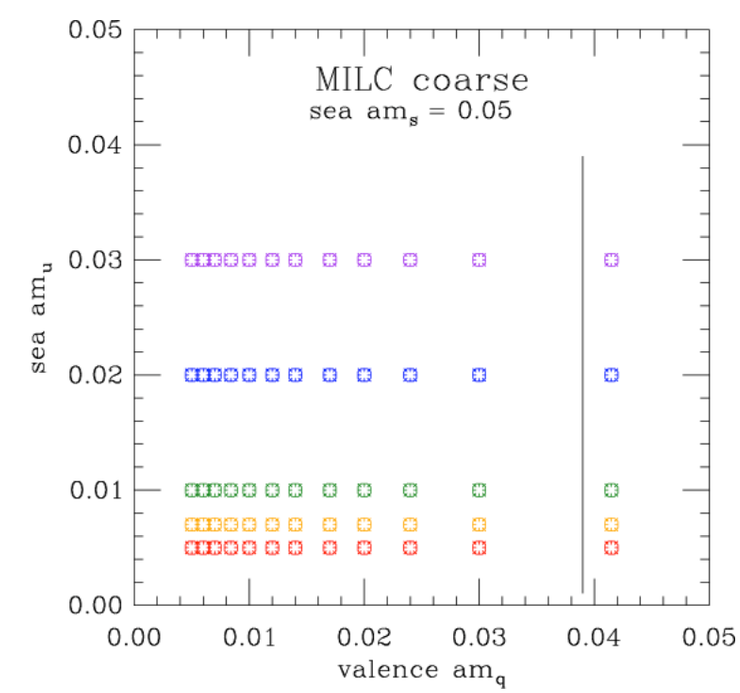
Chiral Extrapolation f_{D_s}



- Interpolate in valence m_q to get down to real m_s.
- Extrapolate in sea m_u to get down to real m_l.



Currently obtained in a separate linear fit.



Preliminary Results

- J. Simone *et al.*, hep-lat/0410030 (Lattice '04)

$$\frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} = 1.20 \pm 0.06 \pm 0.06 \,,$$

$$f_{D_s} = 263^{+5}_{-9} \pm 24 \quad \text{MeV} \,,$$

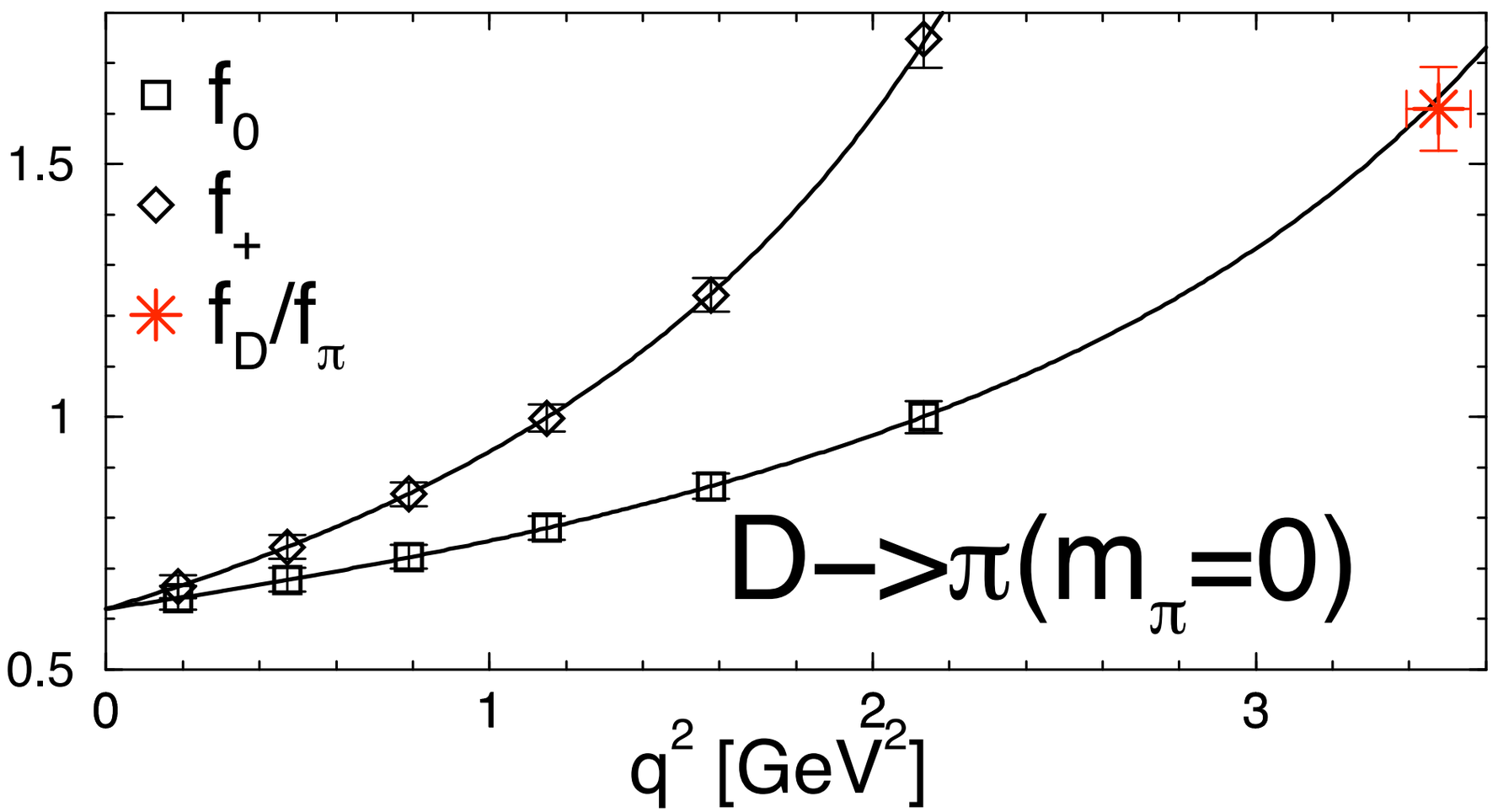
$$f_D = 225^{+11}_{-13} \pm 21 \quad \text{MeV} \,.$$

$f_D = 202 \pm 41 \pm 17$ MeV
CLEO-c, `hep-ex/0411050`

discretization uncertainty as in form factors.

Soft pion theorem

$$f_0^{D\to\pi}(q^2_{\rm max}) = \frac{f_D}{f_\pi}$$



$$\frac{f_D^{\rm LQCD}}{f_\pi^{\rm PDG}}$$

Outlook

- We will combine form factors and decay constants to obtain combinations that can be compared directly to experiment, with no CKM input:

$$\frac{1}{\Gamma_{D\to l\nu}}\frac{d\Gamma_{D\to\pi l\nu}}{dq^2} \propto \left|\frac{f_+^{D\to\pi}(q^2)}{f_D}\right|^2$$

$$\frac{1}{\Gamma_{D_s\to l\nu}}\frac{d\Gamma_{D\to K l\nu}}{dq^2} \propto \left|\frac{f_+^{D\to K}(q^2)}{f_{D_s}}\right|^2$$

B_c

B_c

- Meson composed of a beautiful anti-quark and a charmed quark.
- Unusual beast
  - contrast with B_s & D_s, ψ & Υ: v_c = 0.7.
  - no annihilation to gluons
- Prediction: α_s, m_b, m_c taken from bottomonium and charmonium

Heavy Quarks

- With heavy quarks in lattice QCD there are two short distances: a, $1/m_Q$
- Use effective field theories to control them: short-distance coefficients $C_i(m_Q a)$.
  - Eichten; Lepage *et al.*; El-Khadra, ASK, PBM
- Use latNRQCD for *b* and Fermilab for *c*.

Lattice NRQCD

$$\mathcal{L}_{\mathrm{QCD}}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{HQ}}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{LGT}}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{HQ}(a)}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{HQ}}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{QCD}}$$

Fermilab Method

$$\mathcal{L}_{\mathrm{QCD}}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{LGT}}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{HQ}(a)}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{HQ}}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{QCD}}$$

Essentials

- We calculate two mass splittings

$$\Delta_{\psi\Upsilon} \;=\; m_{B_c} - \tfrac{1}{2}(\bar{m}_\psi + m_\Upsilon) \quad \text{quarkonium baseline}$$

$$\Delta_{D_s B_s} \;=\; m_{B_c} - (\bar{m}_{D_s} + \bar{m}_{B_s}) \quad \text{heavy-light baseline}$$

- Everything is gold-plated, in the sense that the mesons are all stable, and far from threshold.

- Chiral extrapolations mild.

Error Analysis

- Statistical error is straightforward & small.
- Uncertainty from a^{-1}, m_b, m_c easy to propagate: latter two are ± 10, ± 5 MeV.
- Main problem is to estimate the discretization effect for the heavy quarks

Discretization Effects

(short distance mismatch) • (matrix element)

- Use calculations of tree-level mismatches
- Wave hands for one-loop mismatches
- Estimate matrix elements in potential models
- *Check* framework with other calculations

Results

- Splittings:

$$\begin{aligned}\Delta_{\psi\Upsilon} &= 39.8 \pm 3.8 \pm 11.2^{+18}_{-\ 0}\ \text{MeV},\\ \Delta_{D_s B_s} &= -\left[1238 \pm 30 \pm 11^{+\ 0}_{-37}\right]\ \text{MeV},\end{aligned}$$

- Meson mass:

$$\begin{aligned}m_{B_c} &= 6304 \pm\ \ 4 \pm 11^{+18}_{-\ 0}\ \text{MeV},\\ m_{B_c} &= 6243 \pm 30 \pm 11^{+37}_{-\ 0}\ \text{MeV},\end{aligned}$$

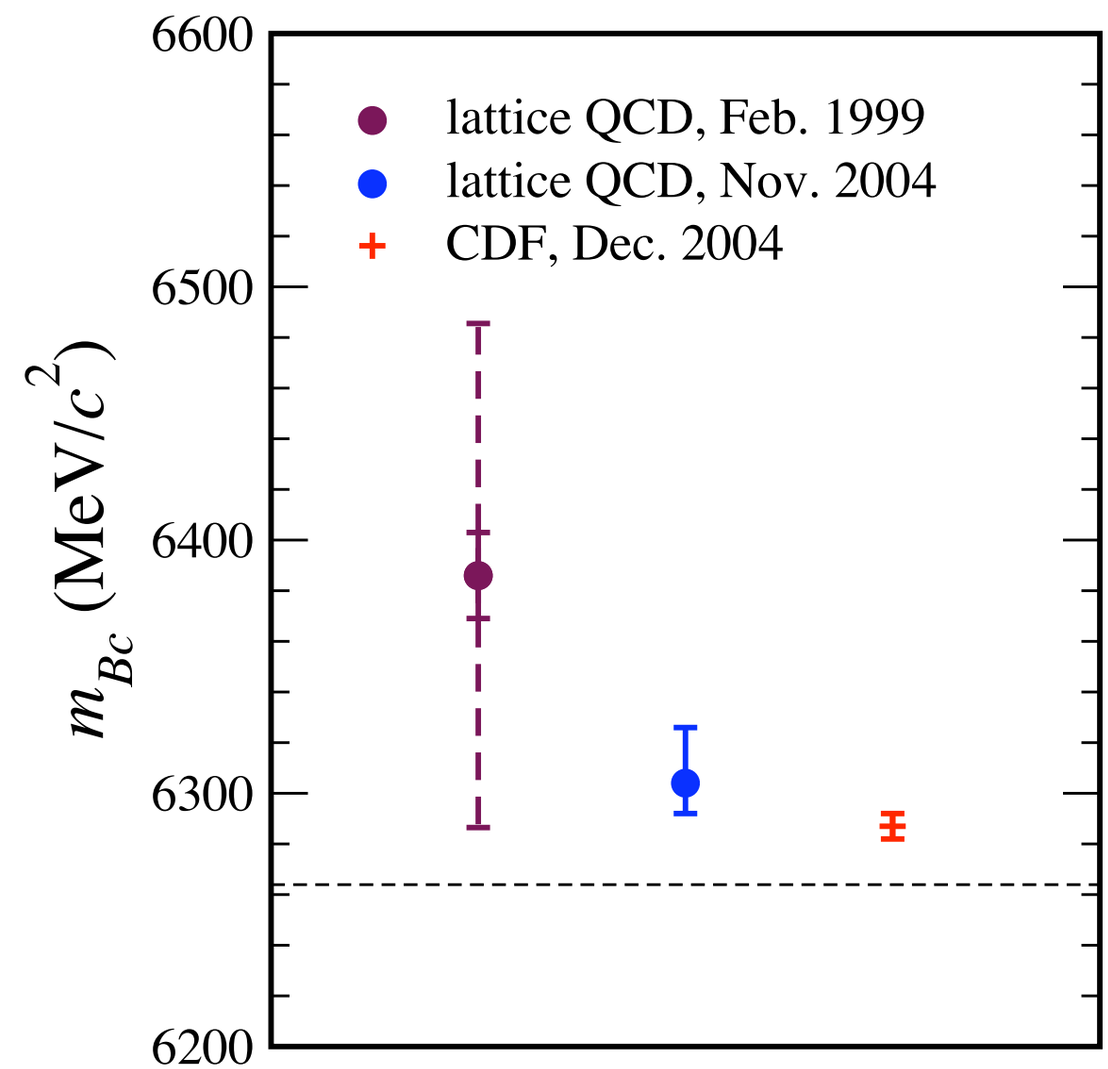
- More checks on quarkonium baseline, so it is our main result.

Comparison

$m_{B_c}^{n_f=0} = 6386 \pm 9 \pm 15 \pm 98$ MeV
[Phys. Lett. B 453, 289 (1999)]

$m_{B_c}^{2+1} = 6304 \pm 4 \pm 11^{+18}_{-0}$ MeV
[`hep-lat/0411027` → PRL]

$m_{B_c}^{\text{expt}} = 6287 \pm 5$ MeV
[CDF, W&C seminar, 12/3/2004]
hep-ex/0505076



Summary

Achievements

- Results for leptonic and semi-leptonic *D* decays and the mass of the B_c meson.
- Agreement with BES, CLEO, FOCUS, and CDF with similar time-scale and error, including *pre*dictions.
- Tests heavy-quark methods (B_c), light valence quarks (f_D), and light sea quarks (f_+).
- Methods now in use for *B* physics.

Junior Collaborators

- Masataka Okamoto, semi-leptonic decays
- Chris Aubin, staggered chiral PT
- Matthew Nobes, pQCD matching
- Ian Allison, B_c mass

Big Shots

- Thanks to MILC for making its ensembles freely available.
- Thanks to SciDAC and Fermilab for funding the clusters used for the analysis.
- Thanks to USQCD (*i.e.*, you) for granting time on the cluster.